Performance Analysis of SVPWM Inverter Fed Permanent Magnet Synchronous Motor

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Abstract—In this paper a space vector pulse width modulation (SVPWM) technique based on mathematical mapping is proposed to reduce total harmonic distortion (THD) of inverter fed permanent magnet synchronous motor (PMSM). Voltage space vectors are studied and simulation of an inverter fed permanent magnet synchronous motor is realized. Afterwards a low pass filter is attached to simulation system to suppress the harmonics generated in the system. The basic purpose of voltage vector selection is to facilitate look up table method. In this study, an inverter switching table is mapped for the permanent magnet synchronous motor (PMSM) control which is performed by selecting the appropriate stator space vectors from the table. Space vector pulse width modulation (SVPWM) algorithm uses a simple mapping to generate gate signals for inverter. The implemented space vector pulse width modulation (SVPWM) control is used to achieve better dynamic performance response with lower total harmonic distortion.

Index Terms—MATLAB/ Simulink, PMSM, SVPWM Inverter, Total Harmonic Distortion (THD).

I. INTRODUCTION

During last three decades AC machine drives are becoming more and more popular, especially Permanent Magnet Synchronous Machine (PMSM) has been used in many automation fields such as robot, metal cutting machines, precision machining, etc. The PMSM drives are ready to meet sophisticated requirements such as fast dynamic response, high power factor, and wide operating speed range like high performance applications, as a result, a gradual gain in the use of PMSM drives will surely be in the future market in low and mid power applications. Recently, PMSM is applied to traction drive for electric vehicles and railway vehicles [2-3]. Fig.1 shows the block diagram of voltage source inverter (VSI) fed PMSM Drive.

The space vector modulation technique is somewhat similar to the sine+3rd harmonic PWM technique but the method of implementation is different. The concept of voltage space-vector, in analogy with the concept of flux space-vector as used in three-phase ac machine. The stator windings of a 3-phase AC machine (cylindrical rotor), when fed with a three-phase balanced current produce a resultant flux space-vector that
rotates at synchronous speed in the space. The flux vector is oriented along the axis of individual phase winding. The magnitude of the flux alternating in nature as the current flowing through it. The magnitude of the resultant flux due to all three windings is, however, fixed at 1.5 times the peak magnitude due to individual phase windings. The resultant flux is commonly known as the synchronously rotating flux vector.

The SVPWM control scheme, the switching duties of the inverter switches are determined by calculating the required voltages forcing the motor phase currents to follow corresponding references. If the motor and inverter parameters are well known, the SVPWM inverter shows fast transient response and no steady state error [3].

At very low frequencies, due to the low value of the stator voltage and the effects of the decrease of the ohmic voltage and the inverter component voltage decreases, the error of the integration function increases [5]. In order to avoid the shift of a pure integration in voltage model, a low pass (LP) filter integrator is used. LP filter eliminates the shifting but causes phase and amplitude errors. Therefore, the driver performance decreases especially at frequencies close to the cut-off frequency of the filter. Further study is available to improve the flux estimation using LP filter. A purposed method [4], utilizes an adaptive control system that depends on the force and the stator flux being orthogonal.

II. PMSM MODELING

Following assumptions are considered before establishing the mathematical modeling of permanent magnet synchronous motor:

i. Neglects the saturation of the electric motor ferrite core.

ii. Neglects turbulent flow and hysteresis loss in electric motor.
iii. The current in electric motor is symmetrical three phase sinusoidal current.

The mathematical model is similar to that of the wound rotor synchronous motor. Since there is no external source connected to the rotor side and variation in the rotor flux with respect to time is negligible, there is no need to include the rotor voltage equation. Rotor reference frame is used to derive the modal of the PMSM [9].

The electrical dynamic equation in terms of phase variable can be written as:

\[ V_A = R_A i_A + p\psi_A \] ........................................ (1)

\[ V_B = R_B i_B + p\psi_B \] ........................................ (2)

\[ V_C = R_C i_C + p\psi_C \] ........................................ (3)

Where \( V_A, V_B \) and \( V_C \) are instantaneous phase voltage, \( i_A, i_B \) and \( i_C \) are instantaneous phase current, \( R_A \) is phase resistant, \( p \) is derivative operator, \( \psi_A, \psi_B \) and \( \psi_C \) are rotor coupling flux linkage.

While the flux linkage equation are:

\[ \psi_A = L_A i_A + \psi, \cos \theta \] ........................................ (4)

\[ \psi_B = L_B i_B + \psi, \cos \left( \theta - \frac{2\pi}{3} \right) \] ........................................ (5)

\[ \psi_C = L_C i_C + \psi, \cos \left( \theta + \frac{2\pi}{3} \right) \] ........................................ (6)

Where, \( L_A \) is phaseInductance.

The transformation from 3-phase to 2-phase quantities can be written in matrix form as:

\[
\begin{bmatrix}
  V_d \\
  V_q \\
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & \sin \theta \\
  -\sin \theta & \cos \theta \\
\end{bmatrix}
\begin{bmatrix}
  V_a \\
  V_b \\
\end{bmatrix}
\] ........................................ (8)

According to above transformation, \( d-q/abc \) Transformation may be written as:

\[
\begin{bmatrix}
  V_d \\
  V_q \\
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & -\sin \theta \\
  -\sin \left( \theta - \frac{2\pi}{3} \right) & -\sin \left( \theta + \frac{2\pi}{3} \right) \\
\end{bmatrix}
\begin{bmatrix}
  V_a \\
  V_b \\
\end{bmatrix}
\] ........................................ (9)

Simple transformed equation are:

\[ V_d = R_S i_d + pL_q i_d + \omega \psi_q \] ........................................ (10)

\[ V_q = R_S i_q + pL_q i_q + \omega \psi_q \] ........................................ (11)

Where, \( L_d \) and \( L_q \) are called \( d \)-and \( q \)-axis synchronous inductance respectively, \( \omega \) is motor electrical speed.

Where, \( V_a \) and \( V_b \) are orthogonal space phasor.

The park transformation in matrix form can be represented as:

\[
\begin{bmatrix}
  V_a \\
  V_b \\
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & \sin \theta \\
  -\sin \theta & \cos \theta \\
\end{bmatrix}
\begin{bmatrix}
  V_a \\
  V_b \\
\end{bmatrix}
\] ........................................ (8)

Fig.3 Vector Diagram of PMSM Motor

\[
\begin{bmatrix}
  V_d \\
  V_q \\
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & -\sin \theta \\
  -\sin \left( \theta - \frac{2\pi}{3} \right) & -\sin \left( \theta + \frac{2\pi}{3} \right) \\
\end{bmatrix}
\begin{bmatrix}
  V_a \\
  V_b \\
\end{bmatrix}
\] ........................................ (9)

Simple transformed equation are:

\[ V_d = R_S i_d + pL_q i_d + \omega \psi_q \] ........................................ (10)

\[ V_q = R_S i_q + pL_q i_q + \omega \psi_q \] ........................................ (11)

Where, \( L_d \) and \( L_q \) are called \( d \)-and \( q \)-axis synchronous inductance respectively, \( \omega \) is motor electrical speed.
The produced torque $T_e$ which is power divided by mechanical speed can be represented as:

$$T_e = \frac{3}{2} p_n \left( \psi_i q_d + (L_d - L_q)i_q i_q \right)$$ ..............(12)

Where, $p_n$ is pole logarithm.

It is apparent from the above equation that the produced torque is composed of two distinct mechanisms. The first term corresponds to "the mutual reaction torque" occurring between $i_q$ and the permanent magnet, while the second term corresponds to "the reluctance torque" due to the difference in $d$- and $q$-axis reluctance [9]. Note that $L_d = L_q = L$ for the motor, so an expression for the torque generated by a PMSM is:

$$T_e = \frac{3}{2} p_n \psi_i q_d$$ ......................(13)

In the presence of a $d$-axis stator current, the $d$-axis and $q$-axis currents are not decoupled, and the model is nonlinear. It can be shown in the torque Eq.(12). Under the assumption that id= 0, the system becomes linear and resembles. Thus vector control of PMSM provides approximate desired dynamic characteristics.

In general, the mechanical equation of the PMSM can be represented as

$$T_e = J_M \omega_M + T_d + B_M \omega_M$$ .............(14)

Where,

$\omega_M = \text{rotor angular speed},$

$J_M = \text{motor moment inertia constant},$

$B_M = \text{damping coefficient},$

$T_d = \text{torque of the motor external load disturbance},$

$T_e = \text{electromagnetic torque}.$

III. SPACE VECTOR PULSE WIDTH MODULATION (SVPWM)

The SVPWM consists of four major processes:

i. Sector Identification,

ii. Vector action time,

iii. Computation of switching time

iv. Generation of PWM,

Fig.5 shows the process of generation of SVPWM and Fig.6 shows the space vector of three-phase voltage source inverter (VSI) divided into six sectors based on the six fundamental vectors $V_x$ (x=1, 2, 3…, 6). Any voltage vectors in this vector space can be synthesized by two fundamental vectors $V_x$ and $V_{x+1}$. For example the voltage vector in sector I can be represented as a combination of active vectors $V_1$ and $V_2$. Within a switching cycle $T_s$, the components for each fundamental vector $V_x$ is related to the occupied time $T_n$ and unoccupied time of the null vectors. The locus of the maximum $V_s$ is represented by the envelope of the hexagon formed by the basic space vectors. Thus the magnitude of $V_s$ must be limited to the shortest radius of this envelope when $V_s$ is revolving, which gives a maximum magnitude of $\frac{V_s}{\sqrt{2}}$ for $V_s$. 

![Fig.5 Block Diagram of SVPWM](image)

![Fig.6 Sectors of SVPWM](image)
For the computation of sector and vectors, the three phase a-b-c voltage is transformed to α-β reference frame using the Clarke transformations. In Field oriented control of PMSM the α-β voltages are obtained from the d-q voltages.

A. Sector Identification

To determine the switching time instants and switching sequence, it is important to know the sector in which the reference vector lies. Following algorithm can be used to determine the sector of the reference output voltage vector. Three intermediate variables are considered as $V_{ref1}$, $V_{ref2}$ and $V_{ref3}$ [10]

A, B and C are considered as logical variables which takes the values 0 or 1 depending on the conditions:

If $V_{ref1}>0, \quad A=1$ else $A=0$……………….. (17)

If $V_{ref2}>0, \quad B=1$ else $B=0$……………….. (18)

If $V_{ref3}>0, \quad C=1$ else $C=0$……………….. (19)

Using the logical variables A, B and C, the variable N is identified as:

$$N=A+2B+4C$$

……… (20)

Values of N is used to map the sector (P) where the vector lies using the Table 1.

TABLE 1. MAPPING OF N TO P

<table>
<thead>
<tr>
<th>N</th>
<th>3</th>
<th>1</th>
<th>5</th>
<th>4</th>
<th>6</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

B. Calculation of Action Time $T_1$ and $T_2$ of Basic Voltage Vector

The action time of two adjacent basic vector in a certain sector is defined as $t_1$ and $t_2$. In traditional SVPWM algorithm, space angles and trigonometric functions are used to calculate the values of $t_1$ and $t_2$, which makes the process complex. In this method these values are calculated using the $V_a$ and $V_b$. Applying the volt-second balance principle to the orthogonal decomposition rates of the basic vectors, $t_1$ and $t_2$ can be mapped from Table 2.

Where; X, Y and Z are given in [10]

TABLE 2. MAPPING X, Y, Z TO $T_1$ AND $T_2$

<table>
<thead>
<tr>
<th>sector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>-Z</td>
<td>Y</td>
<td>X</td>
<td>Z</td>
<td>-Y</td>
<td>-X</td>
</tr>
<tr>
<td>$T_2$</td>
<td>X</td>
<td>Z</td>
<td>-Y</td>
<td>-X</td>
<td>-Z</td>
<td>Y</td>
</tr>
</tbody>
</table>

C. Determination of $T_a$, $T_b$ and $T_c$:

$T_a$, $T_b$ and $T_c$ correspond to the time comparison values of each phase. Intermediate variables $T_{a-on}$, $T_{b-on}$ and $T_{c-on}$ are used to map the comparison values from Table 3:

TABLE 3. OPERATION OF $T_a$, $T_b$ AND $T_c$

<table>
<thead>
<tr>
<th>N</th>
<th>3</th>
<th>1</th>
<th>5</th>
<th>4</th>
<th>6</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_a$</td>
<td>Ta-on</td>
<td>Tb-on</td>
<td>Tc-on</td>
<td>Tc-on</td>
<td>Tb-on</td>
<td>Ta-on</td>
</tr>
<tr>
<td>$T_b$</td>
<td>Tb-on</td>
<td>Ta-on</td>
<td>Tb-on</td>
<td>Tc-on</td>
<td>Tb-on</td>
<td>Tc-on</td>
</tr>
<tr>
<td>$T_c$</td>
<td>Tc-on</td>
<td>Tc-on</td>
<td>Tc-on</td>
<td>Ta-on</td>
<td>Ta-on</td>
<td>Tc-on</td>
</tr>
</tbody>
</table>

TABLE 4. SVPWM VOLTAGE

<table>
<thead>
<tr>
<th>Vector</th>
<th>$V_{ab}$</th>
<th>$V_{bc}$</th>
<th>$V_{ca}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V0={000}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$V1={100}$</td>
<td>+$\text{Vdc}$</td>
<td>0</td>
<td>-$\text{Vdc}$</td>
</tr>
<tr>
<td>$V2={110}$</td>
<td>0</td>
<td>+$\text{Vdc}$</td>
<td>-$\text{Vdc}$</td>
</tr>
<tr>
<td>$V3={010}$</td>
<td>-$\text{Vdc}$</td>
<td>+$\text{Vdc}$</td>
<td>0</td>
</tr>
<tr>
<td>$V4={011}$</td>
<td>-$\text{Vdc}$</td>
<td>0</td>
<td>+$\text{Vdc}$</td>
</tr>
<tr>
<td>$V5={001}$</td>
<td>0</td>
<td>-$\text{Vdc}$</td>
<td>+$\text{Vdc}$</td>
</tr>
<tr>
<td>$V6={101}$</td>
<td>+$\text{Vdc}$</td>
<td>-$\text{Vdc}$</td>
<td>0</td>
</tr>
<tr>
<td>$V7={111}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In sector $V0 \{000\}$ and $V7 \{111\}$ state no voltage generated because of a short circuit situation is created.

IV. SIMULATION RESULTS AND DISCUSSION

A. Parameter of PMSM

Permanent magnet synchronous motor parameters are given in table 5.
TABLE 5. PARAMETER OF PMSM

<table>
<thead>
<tr>
<th>Name of parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Pole Pair (p)</td>
<td>4</td>
</tr>
<tr>
<td>Stator Resistance</td>
<td>0.9585Ω</td>
</tr>
<tr>
<td>(d)-axis inductance (L_d)</td>
<td>0.00835H</td>
</tr>
<tr>
<td>(q)-axis inductance (L_q)</td>
<td>0.00835H</td>
</tr>
<tr>
<td>Flux Linkage</td>
<td>0.01827 V.s</td>
</tr>
<tr>
<td>Inertia</td>
<td>0.0046329 kg.m²</td>
</tr>
<tr>
<td>Viscous Damping</td>
<td>0.0003035N-m.s</td>
</tr>
<tr>
<td>Rotor Type</td>
<td>Round Rotor</td>
</tr>
</tbody>
</table>

Fig.7 shows the simulation diagram of SVPWM inverter fed permanent magnet synchronous motor drive. Fig.8 shows the electromagnetic torque of the Permanent magnet synchronous drive. From the figure it may be observed that Torque transient in the PMSM system is present form 0sec-0.11sec. After 0.11 sec torque become stabilize and reaches at steady state.

Fig.8 Electromagnetic Torque

Fig.9 Speed Response

Fig.9 shows the speed response of the SVPWM inverter fed permanent magnet synchronous motor. The figure shows the speed transient of system. The speed reaches at its steady state at 0.11sec its shows better dynamic performance of system as comparison to other conventional method.
Fig.10 shows the three phase stator current of SVPWM inverter fed permanent magnet synchronous motor. It may be observed from the figure that there is a distortion in the three phase current at the starting of the motor but after one cycle stator current get stabilized.

The input voltages $V_{an}$, $V_{bn}$, $V_{cn}$, of permanent magnet synchronous motor have been shown in fig.11. In fig.12 frequency analysis of neutral voltage is shown, which shows the total harmonic distortion 141%, 141.01%, 141.02% $V_{an}$, $V_{bn}$, $V_{cn}$, respectively. The SVPWM inverter voltage source is 400V DC.

This neutral voltage is filtered by low pass filter which is used in the system for minimizing the total harmonic distortion of the SVPWM voltage source inverter. Fig.13 (a), (b), (c) show the filtered output voltage with a minimum total harmonic distortion. Spectrum analysis of filtered voltage is shown in the fig.14 (a), (b), (c) respectively. The total harmonic distortion is 1.28%, 1.47%, 1.29% $V_{an}$, $V_{bn}$, $V_{cn}$, respectively.
Fig. 13 SVPWM Inverter Filtered Voltage
(a) Van, (b) Vbn, (c) Vcn

Fig. 14 SVPWM Inverter Filtered Voltage
FFT Analysis
(a) Van, (b) Vbn, (c) Vcn

Fig. 15 Stator Current Ia, Ib, Ic,
Fig. 15 (a), (b), (c), show the stator current Ia, Ib, Ic, of permanent magnet synchronous motor and fig.16 shows the frequency domain analysis these figure show that the total harmonic distortion in phase a, b, c, are 1.38%, 1.23%, 1.50%.
V. CONCLUSIONS

In the present work a space vector pulse width modulation algorithm has been developed for improving the performance of inverter fed permanent magnet synchronous motor. A simulation model of SVPWM voltage source inverter (VSI) fed PMSM using the optimized Simulink blocks has been presented. The SVPWM method shows its superior performance characteristics such as low total harmonic distortion (THD) as compare to other pulse width modulation technique (sinusoidal pulse width modulation, sine+3rd harmonic modulation etc.). The total harmonic distortion presented in the SVPWM inverter voltage is 1.28%, 1.47%, 1.29%. Hence the total harmonic distortion is low as compare to other pulse width modulation technique.

REFERENCES


