Uncertainty and Disturbance Estimator Based Sliding Mode Control of 2nd Order System

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ABSTRACT
This work deals with the issue of control of an uncertain system by using uncertainty and disturbance estimator (UDE). A typical second order system is considered for illustration. The results may be extended to a real life application. UDE is used to estimate the uncertainty and the control law ensures robust performance. The proposed technique is validated through simulation in MATLAB. The effectiveness of UDE based control is shown for different types of disturbances.

Keywords
Uncertainty and disturbance estimator (UDE), Robust control, Disturbance estimation.

I. INTRODUCTION
In most of the practical systems uncertainties are always present. Uncertainty includes unmodeled dynamics and external disturbances. Disturbance signals and dynamic perturbations are two varieties of uncertainties. Input and output disturbance, sensor noise and actuator noise, etc are included in disturbance signals. A mathematical model of any real system is always just an approximation of the real physical system. The difference between actual and mathematical model includes unmodeled dynamics (usually high-frequency), neglected nonlinearities in the modeling, effects of reduced-order models, and system parameter variations due to environmental changes. The stability and performance of a control system may adversely be affected due to these uncertainties. With growing interest in high-precision control, utilization of disturbance rejection technique is generally required in the controller design. During the past decades, Time delay control (TDC) [3] is used for system with unknown dynamics. It is based on the assumption that a continuous signal remains unchanged during a small enough period, the past observation of uncertainties and disturbances is used to modify the control action directly. But it has some limitations such as, presence of oscillations in control system and due to delay system becomes unstable.

To overcome this issue recently an uncertainty and disturbance estimator (UDE)-based control method was proposed in [1]. It has good capability of uncertainty and disturbance rejection and reference tracking. UDE is considered as replacement of time delay control (TDC). UDE-based control method is based on assumption that a continuous signal can be approximated when it passes through an appropriate filter. Notable feature of UDE is that it is not affected by modelling inaccuracies and does not require apriori knowledge of disturbances, except the information about the bandwidth, during the design process (but needed for the analysis of stability).

Disturbances can be external disturbances or internal parameter variations. UDE based control law can effectively tackle all such uncertainties. UDE technique is successfully applied to diverse system like non-affine nonlinear system [4], robust input-output linearization [5], robot manipulator [7], voltage control of DC-DC power converter [8], robust control of electric motor drive [9], control of Unmanned Aerial vehicles.
(UAVs) [10], current control scheme for PMSM drives[11], robust control of single axis gimbal platform for micro air vehicles (MAV) [12], trajectory tracking control of piezoelectric stages [13] and nonlinear state delay system [14]. This paper demonstrates the usefulness of the elegant technique by considering a simple academic example. However, the results may be obtained for complex system as well.

The paper is organized as follows: section II presents derivation of UDE based control law. The performance of proposed control scheme is demonstrated through simulation results in section III. And finally section IV is the conclusion.

II. UDE-BASED CONTROL LAW

A. System description

The $n^{th}$ order system under conditions of uncertainty and disturbance can be expressed as

$$
\dot{X} = (A + \Delta A)X + (B + \Delta B)u + \dot{d}
$$

(1)

$$
\dot{X} = A + B + B (X,t)
$$

(2)

Where, $X$ is state variable $(n \times 1)$, $u$ is control input, $A$ is known state matrix $(n \times n)$, $B$ is known input matrix $(n \times 1)$. Here $\Delta A$ is uncertainty in plant matrix and $\Delta B$ is uncertainty in input matrix. $D(X, t)$ is the lumped disturbance.

B. Control design

The sliding surface equation is,

$$
\sigma = C^T * X
$$

where, $C^T$ is coefficient matrix.

Now,

$$
\dot{\sigma} = C^T * \dot{X}
$$

From equation (2)

$$
\dot{\sigma} = C^TAX + C^TBu + C^TBD(X, t)
$$

(3)

Control input $u$ is splits in two term $u_e$ and $u_h$

Therefore equation (3) become,

$$
\dot{\sigma} = C^TAX + C^TBu_e + C^TBu_h + C^TBD(X, t)
$$

(4)

From equation (4),

$$
u_e = -(C^TB)^{-1}C^TAX
$$

(5)

After putting equation (5) in (4) we get,

$$
\dot{\sigma} = C^TAX - C^TB[(C^TB)^{-1}C^TAX] + C^TBu_h + C^TBD(X, t)
$$

$$
D(X, t) = (C^TB)^{-1}\dot{\sigma} - u_h
$$

(6)
In other words, the unknown dynamics and the disturbances can be observed by the system states and the control signal. However, it cannot be used in the control law directly. Assume that \( G(s) \) is a strictly proper low-pass filter with unity steady-state gain and broad enough bandwidth, then, \( D(X, t) \) can be accurately approximated by,

\[
\bar{D}(X, t) = G(s)\{D(X, t)\}
\]

Select

\[
u_n = -\bar{D}(X, t) - (C^TB)^{-1}K\sigma
\]

From equation (6) above equation rewritten as,

\[
u_n = -(C^TB)^{-1}\dot{\sigma} - u_pG_f(s) - (C^TB)^{-1}K\sigma
\]

\[
u_n = -(C^TB)^{-1}\dot{\sigma}G_f(s) - \frac{(C^TB)^{-1}K\sigma}{1 - G_f(s)}
\]

Assume that the frequency range of the system dynamics and the external disturbance is limited by \( \omega_f \) then , \( G_f(s) \) can be chosen as a first order low-pass filter,

\[
G_f(s) = \frac{1}{1 + s\tau}
\]

Where, \( \tau \) is first order filter time constant .

\[
\frac{1}{1 - G_f(s)} = \frac{1}{s\tau} + 1
\]

And now ,

\[
\frac{G(s)}{1 - G_f(s)} = \frac{1}{s\tau}
\]

which means that an integral action is included in the controller.

Considering (10) and (11), the UDE-based control law (9) can be rearranged as

\[
u_n = -(C^TB)^{-1}\dot{\sigma} \frac{1}{s\tau} - (C^TB)^{-1}K\sigma\frac{1}{s\tau} + 1
\]

\[
u_r = -(C^TB)^{-1}\frac{\sigma}{\tau} - (C^TB)^{-1}K\frac{\sigma d}{\tau} - (C^TB)^{-1}K\sigma
\]

The final UDE based control law using (5) and (12) is given as
\[ u = u_e + u_f \]
\[ u = -(C^T B)^{-1} C^T A x - (C^T B)^{-1} \frac{\sigma}{\tau} - (C^T B)^{-1} K \int \frac{\sigma}{\tau} dt - (C^T B)^{-1} K \sigma \]

III. SIMULATION RESULTS
The simulation was carried out to test the performance of control (13) using UDE for the plant in (2) for 5 seconds using SIMULINK software.

The system matrices are \( A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \), \( B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \), constant \( K = 5 \), sliding surface coefficient matrix \( C^T = \begin{bmatrix} 4 & 1 \end{bmatrix} \) and first order filter time constant \( \tau = 0.01 \). Three cases of disturbance have been considered. These conditions are same for all three cases of disturbance.

Case I: The external disturbance considered is a unit step signal acting on the system at time \( t = 1 \text{sec} \).

Case II: The disturbance is sine wave of amplitude 1 and frequency 5 Hz.

It can be seen from Fig. (1) and (2) that the state \( x_1 \) and state \( x_2 \) goes to zero irrespective of disturbance acting on the system. Fig. (3) shows the disturbance estimation capability of UDE.

Case II: The disturbance is sine wave of amplitude 1 and frequency 5 Hz.
The system performance is tested with a sine wave disturbance of amplitude 1 and frequency 5 Hz. Sine wave disturbance is applied at 2 sec.

Case III: Complex disturbance is considered using signal builder.

It can be seen from the Fig (9), (10) that the state $x_1$ and the state $x_2$ go to 0 inspite of disturbance action on it. The disturbance is estimated and shown in Fig. (11).

All the above simulations show that the UDE based control law is robust to different types of disturbances. However if the disturbances are fast varying then the order of the filter needs to be increased for getting satisfactory results.

**CONCLUSION**

In this paper, the strategy of uncertainty and disturbance estimator (UDE) has been successfully employed against various types of disturbances. The control law thus derived has been validated using simulation on MATLAB.
REFERENCES


